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Where Is the Mind? Constructivist and Sociocultural Perspectives on Mathematical Development

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Currently, considerable debate focuses on whether mind is located in the head or in the individual-in-social-action, and whether development is cognitive self-organization or enculturation into established practices. In this article, I question assumptions that initiate this apparent forced choice between constructivist and sociocultural perspectives. I contend that the two perspectives are complementary. Also, claims that either perspective captures the essence of people and communities should be rejected for pragmatic justifications that consider the contextual relevance and usefulness of a perspective. I argue that the sociocultural perspective informs theories of the conditions for the possibility of learning, whereas theories developed from the constructivist perspective focus on what students learn and the processes by which they do so.

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Two major trends can be identified in mathematics education research during the past decade. The first is the generally accepted view that students actively construct their mathematical ways of knowing as they strive to be effective by restoring coherence to the worlds of their personal experience. The theoretical arguments that underpin this position are primarily epistemological and have been advanced by von Glasersfeld (1984, 1987, 1989a). Empirical support is provided by numerous studies that document that there are significant qualitative differences in the understandings that students develop in instructional situations, and that these understandings are frequently very different from those that the teacher intends (Confrey, 1990; Hiebert & Carpenter, 1992). The acceptance of constructivism can be contrasted with a second trend that emphasizes the socially and culturally situated nature of mathematical activity. At least in the United States, this attempt to go beyond purely cognitive analyses reflects a growing disillusionment with the individualistic focus of mainstream psychology (Brown, Collins, & Duguid, 1989; Greeno, 1991; Schoenfeld, 1987). The theoretical basis for this position is inspired in large measure by the work of Vygotsky and that of activity theorists such as Davydov, Leont'ev, and Galperin (Nunes, 1992). Empirical support comes from paradigmatic studies such as those of Carraher, Carraher, and Schliemann (1985), Lave (1988), Saxe (1991), and Scribner (1984), which demonstrate that an individual's arithmetical activity is profoundly influenced by his or her participation in encompassing cultural practices such as completing worksheets in school, shop-ping in a supermarket, selling candy on the street, and packing crates in a dairy.

These constructivist and sociocultural perspectives at times appear to be in direct conflict, with adherents to each claiming hegemony for their view of what it means to know and learn mathematics (Steffe, in press; Voigt, 1992). Thus, there is currently a dispute over both whether the mind is located in the head[1] or in the individual-in-social-action, and whether mathematical learning is primarily a process of active cognitive reorganization or a process of enculturation into a community of practice (Minick, 1989). Similarly, the issue of whether social and cultural processes have primacy over individual processes, or vice versa, is the subject of intense debate (van Oers, 1990). Further, adherents to the two positions differ on the role that signs and symbols play in psychological development. Constructivists tend to characterize them as a means by which students express and communicate their mathematical thinking, whereas sociocultural theorists typically treat them as carriers of either established mathematical meanings or of a practice's intellectual heritage. In general, the attempts of the two groups of theorists to understand the other's position are confounded by their differing usage of a variety of terms, including *activity, setting, context, task, problem, goal, negotiation,* and *meaning*.

The central focus of this article will be on the assumptions that give rise to an apparent forced choice between the two perspectives. In particular, I will argue that mathematical learning should be viewed as both a process of active individual construction and a process of enculturation into the mathematical practices of wider society. The central issue is then not that of adjudicating a dispute between opposing perspectives. Instead, it is to explore ways of coordinating constructivist and sociocultural perspectives in mathematics education. The particular perspective that comes to the fore at any point in an empirical analysis can then be seen to be relative to the problems and issues at hand.

It should be noted that the apparent conflict between constructivist and sociocultural perspectives is not merely a matter of theoretical contemplation. Instead, it finds expression in tensions endemic to the act of teaching. For ex-

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ample, Ball (1993) observes that "current proposals for educational improvement are replete with notions of 'understanding' and 'community'—about building bridges between the experiences of the child and the knowledge of the expert" (p. 374). She then inquires,

> How do I create experiences for my students that connect with what they now know and care about but that also transcend the present? How do I value their interests and also connect them to ideas and traditions growing out of centuries of mathematical exploration and invention? (p. 375)

Ball's references to students' experiences and to valuing their interests imply a focus on their qualitatively distinct interpretations and on the personal goals that they pursue in the classroom. This, in my terms, implies a view of mathematical learning as active construction. In contrast, her reference to students' mathematical heritage suggests a view of mathematical learning as enculturation. Ball goes on to discuss three dilemmas that arise in the course of her practice as a mathematics teacher. She clarifies that these dilemmas of content, discourse, and community "arise reasonably from competing and worthwhile aims and from the uncertainties inherent in striving to attain them" (p. 373). It would therefore seem that the aims of which she speaks and thus the pedagogical dilemmas reflect the tension between mathematical learning viewed as enculturation and as individual construction.

Comparisons and Contrasts

Sociocultural and constructivist theorists both highlight the crucial role that activity plays in mathematical learning and development. However, sociocultural theorists typically link activity to participation in culturally organized practices, whereas constructivists give priority to individual students' sensory-motor and conceptual activity. Further, sociocultural theorists tend to assume from the outset that cognitive processes are subsumed by social and cultural processes. In doing so, they adhere to Vygotsky's (1979) contention that "the social dimension of consciousness is primary in fact and time. The individual dimension of consciousness is derivative and secondary" (p. 30). From this, it follows that "thought (cognition) must not be reduced to a subjectively psychological process" (Davydov, 1988, p. 16). Instead, thought should be viewed as

> something essentially "on the surface," as something located. . .on the borderline between the organism and the outside world. For thought. . .has a life only in an environment of socially constituted meanings. (Bakhurst, 1988, p. 38)

Consequently, whereas constructivists analyze thought in terms of conceptual processes located in the individual, sociocultural theorists take the individual-in-social-action as their unit of analysis (Minick, 1989). From this latter perspective, the primary issue is that of explaining how participation in social interactions and culturally organized activities influences psychological development.

Sociocultural theorists formulate this issue in a variety of different ways. For example, Vygotsky (1978) emphasized the importance of social interaction with more knowledgeable others in the zone of proximal development and the role of culturally developed sign systems as psychological tools for thinking. In contrast, Leont'ev (1981) argued that thought develops from practical, object-oriented activity or labor. Several American theorists have elaborated constructs developed by Vygotsky and his students, and speak of cognitive apprenticeship (Brown, Collins, & Duguid, 1989; Rogoff, 1990), legitimate peripheral participation (Forman, 1992; Lave & Wenger, 1991), or the negotiation of meaning in the construction zone (Newman, Griffin, & Cole, 1989). In contrast to the constructivist's concern with individual students' conceptual reorganizations, each of these contemporary accounts locates learning in coparticipation in cultural practices. As a consequence, educational implications usually focus on the kinds of social engagements that increasingly enable students to participate in the activities of the expert rather than on the cognitive processes and conceptual structures involved (Hanks, 1991).

In contrast to sociocultural theorists' frequent references to the works of Vygotsky, Leont'ev, and Luria, constructivists usually trace their intellectual lineage to Piaget's genetic epistemology (1970, 1980), to ethnomethodology (Mehan & Wood, 1975), or to symbolic interactionism (Blumer, 1969). As this set of references indicates, it is possible to distinguish between what might be called psychological and interactionist variants of constructivism. Von Glasersfeld's development of the epistemological basis of the psychological variant incorporates both the Piagetian notions of assimilation and accommodation, and the cybernetic concept of viability. Thus, he uses the term knowledge in "Piaget's adaptational sense to refer to those sensory-motor and conceptual operations that have proved viable in the knower's experience" (1992, p. 380). Further, traditional correspondence theories of truth are dispensed with in favor of an account that relates truth to the effective or viable organization of activity: "Truths are replaced by viable models—and viability is always relative to a chosen goal" (1992, p. 384). In this model, perturbations that the cognizing subject generates relative to a purpose or goal are posited as the driving force of development. As a consequence, learning is characterized as a process of self-organization in which the subject reorganizes his or her activity to eliminate perturbations (von Glasersfeld, 1989b). As von Glasersfeld notes, his instrumentalist approach to knowledge is generally consistent with the views of contemporary neopragmatist philosophers such as Bernstein (1983), Putnam (1987), and Rorty (1978).

Although von Glasersfeld defines learning as self-organization, he acknowledges that this constructive activity occurs as the cognizing individual interacts with other members of a community. Thus, he elaborates that knowledge refers to "conceptual structures that epistemic agents, given the range of present experience within their tradition of thought and language, consider viable" (1992, p. 381). Further, he contends that "the most frequent source of perturbations for the developing cognitive subject is interaction with others" (1989b, p. 136). Bauersfeld's interactionist version of constructivism complements von Glasersfeld's psychological focus in that both view communication as a process of mutual adaptation wherein individuals negotiate meanings by continually modifying their interpretations (Bauersfeld, 1980; Bauersfeld, Krummheuer, & Voigt, 1988). However, whereas von

Glasersfeld tends to focus on individuals' construction of their ways of knowing, Bauersfeld emphasizes that "learning is characterized by the subjective reconstruction of societal means and models through negotiation of meaning in social interaction" (1988, p. 39). In accounting for this process of subjective reconstruction, he focuses on the teacher's and students' interactive constitution of the classroom microculture. Thus, he argues that

> participating in the processes of a mathematics classroom is participating in a culture of mathematizing. The many skills, which an observer can identify and will take as the main performance of the culture, form the procedural surface only. These are the bricks of the building, but the design of the house of mathematizing is processed on another level. As it is with culture, the core of what is learned through participation is *when* to do what and *how* to do it. . . .The core part of school mathematics enculturation comes into effect on the meta-level and is "learned" indirectly. (in press)

Bauersfeld's reference to indirect learning clarifies that the occurrence of perturbations is not limited to those occasions when participants in an interaction believe that communication has broken down and *explicitly negotiate* meanings. Instead, for him, communication is a process of often *implicit negotiations* in which subtle shifts and slides of meaning occur outside the participants' awareness (cf. Cobb & Yackel, in press). In taking this approach, Bauersfeld uses an interactionist metaphor and characterizes negotiation as a process of mutual adaptation in the course of which the teacher and students establish expectations for others' activity and obligations for their own activity (cf. Cobb & Bauersfeld, in press; Voigt, 1985). By way of contrast, Newman et al. (1989), speaking from the sociocultural perspective, define negotiation as a process of mutual appropriation in which the teacher and students continually coopt or use each others' contributions. Here, in line with Leont'ev's (1981) sociohistorical metaphor of appropriation, the teacher's role is characterized as that of mediating between students' personal meanings and culturally established mathematical meanings of wider society. From this point of view, one of the teacher's primary responsibilities when negotiating mathematical meaning with students is to appropriate their actions into this wider system of mathematical practices. Bauersfeld, however, takes the local classroom microculture rather than the mathematical practices institutionalized by wider society as his primary point of reference when he speaks of negotiation. This focus reflects his concern with the process by which the teacher and students constitute social norms and mathematical practices in the course of their classroom interactions. Further, whereas sociocultural theorists give priority to social and cultural process, analyses compatible with Bauersfeld's perspective propose that individual students' mathematical activity and the classroom microculture are reflexively related (Cobb, 1989; Voigt, 1992). In this view, individual students are seen as actively contributing to the development of classroom mathematical practices, and these both enable and constrain their individual mathematical activities. Consequently, it is argued that neither an individual student's mathematical activity nor the classroom microculture can be adequately accounted for without considering the other.

It is apparent from this brief summary of the two perspectives that they address different problems and issues. A sociocultural analysis of a classroom episode might both locate it within a broader activity system that takes account of the function of schooling as a social institution and attend to the immediate interactions between the teacher and students (Axel, 1992). This dual focus is explicit in Lave and Wenger's (1991) claim that their "concept of legitimate peripheral participation provides a framework for bringing together theories of situated activity and theories about the production and reproduction of the social order" (p. 47). In general, sociocultural accounts of psychological development use the individual's participation in culturally organized practices and face-to-face interactions as primary explanatory constructs. A basic tenet underpinning this work is that it is inappropriate to single out qualitative differences in individual thinking apart from their sociocultural situation because differences in students' interpretations of school tasks reflect qualitative differences in the communities in which they participate (Bredo & McDermott, 1992).

In contrast, constructivists are typically concerned with the quality of individual interpretive activity, with the development of ways of knowing at a more micro-level, and with the participants' interactive constitution of classroom social norms and mathematical practices. The burden of explanation in constructivist accounts of development falls on models of individual students' cognitive self-organization and on analyses of the processes by which these actively cognizing individuals constitute the local social situation of their development (Cobb, Wood, & Yackel, 1993). Thus, whereas a sociocultural theorist might view classroom interactions as an instantiation of the culturally organized practices of schooling, a constructivist would see an evolving microculture that does not exist apart from the teacher's and students' attempts to coordinate their individual activities. Further, whereas a sociocultural theorist might see a student appropriating the teacher's contributions, a constructivist would see a student adapting to the actions of others in the course of ongoing negotiations. In making these differing interpretations, sociocultural theorists would tend to invoke sociohistorical metaphors such as *appropriation*, whereas constructivists would typically employ interactionist metaphors such as *accommodation* and *mutual adaptation*. Further, whereas sociocultural theorists typically stress the homogeneity of members of established communities and eschew analyses of qualitative differences in individual thinking, constructivists tend to stress heterogeneity and to eschew analyses that single out pregiven social and cultural practices. From one perspective, the focus is on the social and cultural basis of personal experience. From the other perspective, it is on the constitution of social and cultural processes by actively interpreting individuals.

Construction in Social Practice

Against the background of these contrasts between the two perspectives, I now consider possible coordinations between them. In this section, I explore possible complementarities between Rogoff's (1990) analysis of internalization and von Glasersfeld's (in press) discussion of empirical and reflective abstraction. In a subsequent section, I elaborate my argument by focusing on potential relationships

between Saxe's (1991) sociocultural analysis and Steffe, Cobb, and von Glasersfeld's (1988) constructivist analysis. My general strategy in both cases is to tease out aspects of one position that are implicit in the other.

One of the central notions in Vygotsky's account of development is internalization. For example, in his frequently cited general genetic law of cultural development, Vygotsky argued that

> any higher mental function was external and social before it was internal. It was once a social relationship between two people. . . .We can formulate the general genetic law of cultural development in the following way. Any function appears twice or on two planes. . . .It appears first between people as an intermental category, and then within the child as an intramental category. (Vygotsky, 1960, pp. 197–198)

From the constructivist perspective, this account of internalization from the social realm to the internal cognitive realm leads to difficulties because the interpersonal relations that are to be internalized are located outside the child. Researchers can indeed identify patterns of interaction, collective schemes, and so forth when they analyze videorecordings or transcripts. However, a constructivist might follow Blumer (1969) in arguing that people respond to things in terms of the meaning they have for them rather than to constructs that researchers project into their worlds. From this point of view, the problem of explaining how relations that are real for the detached observer get into the experiential world of the child appears intractable.

Rogoff (1990), who is in many ways a follower of Vygotsky, discusses this difficulty in reference to research on social learning and socialization. She notes that, in this research, children are considered to learn by observing or participating with others. "The underlying assumption is that the external lesson [to be learned] is brought across a barrier into the mind of the child. How this is done is not specified, and remains a deep problem for these approaches" (p. 195). In proposing a solution, Rogoff elaborates Vygotsky's notion of internalization by arguing that children are already engaged in a social activity when they actively observe and participate with others. If children are viewed as being in the social activity in this way

> with the interpersonal aspects of *their* functioning integral to the individual aspects, then what is practiced in social interaction is never on the outside of a barrier, and there is no need for a separate process of internalization. (p. 195, italics added)

Here, Rogoff circumvents the need for an internalization process by proposing that the researcher change his or her perspective and focus on what children's interpersonal activity might mean to them. In constructivist terms, this involves a shift in focus to the mathematical meanings and practices that the child considers are shared with others.

Rogoff's point that children are already active participants in the social practice implies that they engage in and contribute to the development of classroom mathematical practices from the outset. Further,

> in the process of participation in social activity, the individual already functions with shared understanding. The individual's use of this shared understanding is not the same as what was constructed jointly; it is an appropriation of the shared understanding by each individual that

reflects the individual's understanding of and involvement in the activity. (Rogoff, 1990, p. 195)

Rogoff's distinction between the individual's use of a shared understanding and the shared understanding that is constructed jointly is closely related to the distinction that a constructivist might make between an individual child's understanding and the taken-as-shared meanings established by the group (Cobb, Perlwitz, & Underwood, in press; Schutz, 1962). It therefore seems reasonable to conclude from Rogoff's treatment of internalization that mathematical learning is a process of active construction that occurs when children engage in classroom mathematical practices, frequently while interacting with others. Significantly, a similar conclusion can be reached when considering von Glasersfeld's (in press) elaboration of Piaget's developmental theory.

Von Glasersfeld develops his view of learning as self-organization by clarifying the distinction that Piaget made between two types of cognitive reorganization, empirical abstraction and reflective abstraction. In doing so, he emphasizes that an empirical abstraction results in the construction of a property of a physical object, whereas the process of constructing mathematical and scientific concepts involves reflective abstraction. He illustrates the notion of empirical abstraction by describing a situation in which someone wants to drive a nail into a wall, but does not have a hammer. After looking around, the person finds a wooden mallet and begins to use this, only to find that the nail goes into the mallet instead of into the wall. Von Glasersfeld argues that, in this scenario, the person assimilates the mallet to her hammering scheme, but then makes an accommodation when things do not go as expected, and a perturbation is experienced. This accommodation involves an empirical abstraction in that it results in the construction of a novel property for the mallet—it is not the sort of thing that can be used to hammer nails into walls.

The interesting feature of this example for my purposes is that hammering is a cultural practice that involves acting with particular cultural artifacts, hammers and nails. The person's hammering scheme can be viewed as the product of active constructions she made in the course of her initiation into this practice. In other words, hammers, nails, and mallets are, for her, cultural tools that she can use for certain purposes. It is against the background of her engagement in this practice of hammering that she makes the empirical abstraction described by von Glasersfeld. This being the case, it seems reasonable to extend the definition of empirical abstraction by emphasizing both that it results in the emergence of novel physical properties and that it occurs as the individual participates in a cultural practice, often while interacting with others. This formulation involves the coordination of perspectives in that the first part, referring as it does to an experienced novelty, is said from the "inside," whereas the second part is said from the "outside" and locates the individual in a cultural practice.

The assumption that individual activity is culturally situated is also implicit in von Glasersfeld's discussion of the construction of mathematical concepts. Here, the notion of reflective abstraction is used to account for the process by which actions are reified and become mental mathematical objects that can themselves be acted upon (cf. Sfard, 1991; Thompson, 1994). For von Glasersfeld, it is by means of re-

flective abstraction that students reorganize their initially informal mathematical activity. Consider, for example, a situation in which the teacher introduces conventional written fraction symbols to record the results of students' attempts to partition objects such as pizzas fairly. Von Glasersfeld stresses that the students can only interpret the teacher's actions within the context of their ongoing activity. Further, the process by which the symbols come to signify the composition and decomposition of fractional units of some type for at least some of the students is accounted for in terms of the reification of partitioning activity via reflective abstraction. As with the example of the mallet, it can be observed that these conceptual reorganizations occur as the students participate in cultural practices. In this case, these are the mathematical practices that the students help to establish in the classroom. The mathematical concepts they each individually construct are relative to and are constrained by their participation in these practices. It can also be noted that the activities from which the students abstract include their interpretations of others' activity and of joint activities (Voigt, 1992). These considerations suggest that in defining reflective abstraction, we should emphasize both that it involves the reification of sensory-motor and conceptual activity and that it occurs while engaging in cultural practices, frequently while interacting with others. As was the case with the characterization of empirical abstraction, this formulation involves the coordination of perspectives.

In comparing Rogoff's and von Glaserfeld's work, it can be noted that Rogoff's view of learning as acculturation via guided participation implicitly assumes an actively constructing child. Conversely, von Glasersfeld's view of learning as cognitive self-organization implicitly assumes that the child is participating in cultural practices. In effect, active individual construction constitutes the background against which guided participation in cultural practices comes to the fore for Rogoff, and this participation is the background against which self-organization comes to the fore for von Glasersfeld.

Coordinating Perspectives

The complementarity between the sociocultural and constructivist perspectives can be further clarified by considering the analyses of arithmetical activity offered by Saxe (1991) and Steffe et al. (1988). In contrast to the majority of sociocultural theorists, Saxe takes an explicitly developmental perspective that focuses on individuals' understandings while simultaneously emphasizing the influence of cultural practices and the use of sign forms and cultural artifacts. He illustrates his theoretical approach by analyzing the body-parts counting system developed by the Oksapmin people of Papua New Guinea.

Saxe explains that "to count as Oksapmin do, one begins with the thumb on one hand, and follows a trajectory around the upper periphery of the body down to the little finger of the opposite hand" (1991, p. 16). With Western contact and the introduction of tradestores, the Oksapmin had to use this indigenous counting system to solve arithmetical problems that did not emerge in traditional life, such as those of adding and subtracting values. In the course of his analysis of the interplay between the Oksapmin's participation in tradestore activities and their construction of mathematical understandings, Saxe identifies

four developmental levels in the evolution of the body-parts counting system. At the least sophisticated level, individuals do not recognize the need to keep track of the second addend when they attempt to add, say, seven and nine coins. As a consequence, they frequently produce an incorrect sum. In contrast, the most sophisticated of the four levels involves the use of a "halved-body strategy" that incorporates a base-10 system linked to the currency. Here, in adding seven and nine coins,

> individuals use the shoulder (10) as a privileged value. In their computation, they may represent the 9 on one side of the body as biceps (9) and 7 on the other side of the body as forearm (7). To accomplish the problem, a tradestore owner might simply "remove" the forearm from the second side. . .and transfer it to the first side where it becomes the shoulder (the 10th). He then "reads" the answer as 10 + 6 or 16. (p. 21)

In sociocultural terms, the Oksapmin's increasingly sophisticated computational strategies can be viewed as cultural forms. An account of development made from this perspective might focus on the extent to which individual Oksapmin participate in the new practice of economic exchange. Such an account would stress that typically only tradestore owners, who have the most experience with economic transactions, use the sophisticated halved-body strategy. In contrast to this view that social and cultural practices drive development, a constructivist analysis might treat the Oksapmin's computational strategies as cognitive forms created by self-organizing individuals. An account of this latter type might focus on the processes by which individual Oksapmin reflectively abstract from and thus reorganize their enumerating activity, thereby creating increasingly sophisticated arithmetical units. Interestingly, it is possible to develop such an account by using the cognitive models of American children's arithmetical development proposed by Steffe as a source of analogies (Steffe et al., 1988).

We have seen that Oksapmin at the least sophisticated level do not recognize the need to keep track of counting. In contrast, Oksapmin at the next level consciously attempt to keep track. This suggests that these Oksapmin view their counting acts as entities that can themselves be counted. In Steffe et al.'s (1988) terms, these acts carry the significance of counting abstract units. This analysis, which is made from the "inside" rather than the "outside," explains why Oksapmin at the initial level do not recognize the need to keep track of counting. They are yet to reify their counting acts, and, as a consequence, body-parts counting as they currently understand it is simply not the kind of activity that can be kept track of.

This analysis can be extended to account for the development of more sophisticated strategies. For example, when the halved-body strategy is used, a body part such as the biceps (9) appears to symbolize not a single unit but the composite of nine abstract units that would be created by counting to the biceps. In Piagetian terminology, counting has been reified via reflective abstraction, and the biceps symbolizes nine experienced as an arithmetical object that can be conceptually manipulated.

Each of the two perspectives, the sociocultural and the constructivist, tells half of a good story, and each can be used to complement the other. For example, consider a sit-

uation in which a young Oksapmin works in a tradestore and eventually learns the halved-body strategy used by the store owner. A sociocultural explanation might talk of the novice appropriating or internalizing a cultural form. As we have seen, an account of this type has difficulty in explaining how a cultural form that is external to the novice is brought across the barrier and becomes a cognitive form. The constructivist analysis circumvents this difficulty by stressing that rather than internalizing a cultural form that appears to be pregiven, the novice reorganizes his or her own activity. Thus, to paraphrase Rogoff (1990), there is nothing to bring across the barrier and, consequently, no need to posit a process of internalization from the sociocultural to the cognitive realm.

By the same token, the sociocultural perspective complements the constructivist perspective by emphasizing that the novice trader reorganizes his or her counting activity while attempting to achieve goals that emerge in the course of his or her participation in the practice of economic exchange (Saxe, 1991). From this point of view, it is readily apparent that both what counts as a problem and what counts as a legitimate solution are highly normative (cf. Solomon, 1989). Thus, both the process of individual construction and its products, increasingly sophisticated conceptual units, are social through and through. Conversely, it can be argued that the various strategies, viewed as cultural forms, are cognitive through and through in that they result from individual Oksapmin's constructive activities. As was the case with the discussion of Rogoff's and von Glasersfeld's analyses, this coordination of perspectives leads to the view that learning is a process of both self-organization and a process of enculturation that occurs while participating in cultural practices, frequently while interacting with others.

Theoretical Pragmatism

The discussion of Rogoff's, von Glasersfeld's, Saxe's, and Steffe's work indicates that sociocultural analyses involve implicit cognitive commitments, and vice versa. It is as if one perspective constitutes the background against which the other comes to the fore. This contention concerning the relationship between the perspectives can be contrasted with the claims made by adherents to each perspective that mind is either in the head or in the individual-in-social-action. Claims of this type reflect essentialist assumptions. In effect, adherents of both positions claim that they have got the mind right—this is what the mind really is, always was, and always will be, independent of history and culture. A perusal of Geertz's (1983) discussion of Western, Arabic, and Indic visions of the self and of community might lead proponents of a particular perspective to question whether theirs is the God's-eye view.

Following Fish (1989), it can be argued that theorizing is itself a form of practice rather than an activity that stands in opposition to practice. The discussion thus far suggests that if we want our practice of theorizing to be reflexively consistent with the theories we develop as we engage in that practice, we have to give up essentialist claims and take a more pragmatic approach. In this regard, Rorty (1983), who uses the metaphor of wielding vocabulary rather than taking a perspective, argues that

the idea that only a certain vocabulary is suited to human

beings or human societies, that only that vocabulary permits them to be "understood," is the seventeenth-century myth of "nature's own vocabulary" all over again. (p. 163)

For Rorty, the various vocabularies we use or the particular perspectives we take are instruments for coping with things rather than ways of representing their intrinsic nature. Here, Rorty follows Dewey and Kuhn in arguing that we should "give up the notion of science traveling towards an end called 'corresponding with reality' and instead say merely that a given vocabulary works better than another for a given purpose" (p. 157). Thus, "to say something is better understood in one vocabulary than another is always an ellipsis for the claim that a description in the preferred vocabulary is most useful for a certain purpose" (p. 162).

The implication of this pragmatic approach for mathematics education, and for education more generally, is to consider what various perspectives might have to offer relative to the problems or issues at hand. In this regard, I suggest that the sociocultural perspective gives rise to theories of the conditions for the possibility of learning (Krummheuer, 1992), whereas theories developed from the constructivist perspective focus on both what students learn and the processes by which they do so. For example, Lave and Wenger (1991), who take a relatively radical position by attempting to avoid any reference to mind in the head, say that "a learning curriculum unfolds in *opportunities for engagement* in practice" (p. 93, italics added). Consistent with this formulation, they note that their analysis of various examples of apprenticeship in terms of legitimate peripheral participation accounts for the occurrence of learning or failure to learn (p. 63). In contrast, a constructivist analysis would typically focus on the ways in which students reorganize their activity as they participate in a learning curriculum, and on the processes by which the curriculum is interactively constituted in the local situation of development. In my view, both these perspectives are of value in the current era of educational reform that stresses both students' meaningful mathematical learning and the restructuring of the school while simultaneously taking issues of diversity seriously. Constructivists might argue that sociocultural theories do not adequately account for the process of learning, and sociocultural theorists might retort that constructivist theories fail to account for the production and reproduction of the practices of schooling and the social order. The challenge of relating actively constructing students, the local microculture, and the established practices of the broader community requires that adherents to each perspective acknowledge the potential positive contributions of the other perspective. In doing so, constructivists would accept the relevance of work that addresses the broader sociopolitical setting of reform. Conversely, sociocultural theorists would acknowledge the pedagogical dilemmas articulated by Ball (1993) when she spoke of attending to both students' interests and understandings, and to their mathematical heritage.

In dispensing with essentialist claims, this pragmatic approach to theorizing instead proposes that the adoption of one perspective or another should be justified in terms of its potential to address issues whose resolution might contribute to the improvement of students' education. Voigt

(1992) offered a justification of this type when he stated that

> personally the author takes the emphasis on the [individual] subject as the starting-point in order to understand the negotiation of meaning and the learning of mathematics in classrooms. . . .The main reason is that concepts like "socialization," "internalization," "initiation into a social tradition," etc. do not (directly) explain what I think is the most important objective of mathematics education. . . .The prominent objective of mathematics education is not that students produce correct solutions to mathematical problems but that they do it insightfully and by reasonable thinking. What on the behavioral level does in fact not make a difference should be an important subjective difference. (p. 10)

Justifications of this type are, of course, open to challenge. For example, a critic might argue that, in certain circumstances, it is more important that students produce correct answers than that they develop insight. This counterargument does not claim that Voigt's chosen perspective fails to capture the essence of mathematical development. Instead, it questions assumptions about educational objectives and, ultimately, about what counts as improvement in students' mathematics education. In general, claims about what counts as improvement reflect beliefs and values about what it ought to mean to know and do mathematics (or science or social studies) in school. These beliefs and values are themselves open to challenge and criticism, thus bringing to the fore the moral and ethical aspects of educational research and theorizing (Nicholls, 1989).

The central claim of this article, that the sociocultural and constructivist perspectives each constitute the background for the other, implies that justifications should explicitly bring the researcher into the picture by acknowledging his or her interpretive activity. Essentialist claims involve a denial of responsibility—it is social reality that dictates the correct theoretical perspective. In contrast, pragmatic justifications reflect the researcher's awareness that he or she has adopted a particular position for particular reasons. From the sociocultural perspective, a justification of this type would explain why it is not necessary to focus on the actively cognizing student for the purposes at hand. Conversely, constructivists would be obliged to explain why it is not necessary to go beyond the box of the classroom for their purposes, while acknowledging that it is appropriate to take a perspective that locates classroom events within a wider sociopolitical setting for other purposes.

This pragmatic approach to theorizing also contends that ways of coordinating perspectives should be developed while addressing specific problems and issues. In addition, the suggestion acknowledges that Ball and other teachers have something interesting to say when they suggest that the tension in teaching between individual construction and enculturation cannot be resolved once and for all. Teachers instead have to act with wisdom and judgment by continually developing ways to cope with dilemmas in particular situations. A similar modus operandi would appear to be appropriate for researchers as we engage in our practice. In place of attempts to subjugate research to a single, overarching theoretical scheme that is posited a priori, we might follow Ball in reflecting on and documenting our attempts to coordinate perspectives as

we attempt to cope with our specific problems. In doing so, we would give up the quest for an acontextual, one-size-fits-all perspective. Instead, we would acknowledge that we, like teachers, cast around for ways of making sense of things as we address the situated problems of our practice.

Notes

[1]The phrase "mind in the head" is used as a metonymy to refer to individualistic accounts of cognition. Following Johnson (1987) and Varela, Thompson, and Rosch (1991), the phrase should not be read as implying a mind-body dualism.

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